WOLLO UNIVERSITY

KOMBOLCHA INSTITUTE OF TECHNOLOGY

SCHOOL OF CIVIL,ARCHITECTURE AND WATER ENGINEERING

REINFORCED CONCRETE STRUCTURE-II

CHAPTER THREE: SHORT COLUMNS

TARGET GROUP WRIE THIRD YEAR.

Introduction

- A column is a vertical structural member transmitting axial compression loads with or without moments.
- The cross sectional dimensions of a column are generally considerably less than its height.
- Column support mainly vertical loads.
- A column is a special case of a compression member that is vertical.

Classification of column

A. Classification on the basis of geometry; rectangular, square, circular, L-shaped, T shaped, etc. depending on the structural or architectural requirements.

B. Classification on the basis of composition; RC colums, composite columns, in-filled columns, etc.

C. Classification on the basis of lateral reinforcement; tied columns, spiral columns.

D. Classification on the basis of manner by which lateral stability is provided to the structure as a whole; braced columns, un-braced columns.

Cont…

E. Classification on the basis of sensitivity to second order effect due to lateral displacements; sway columns, non-sway columns.

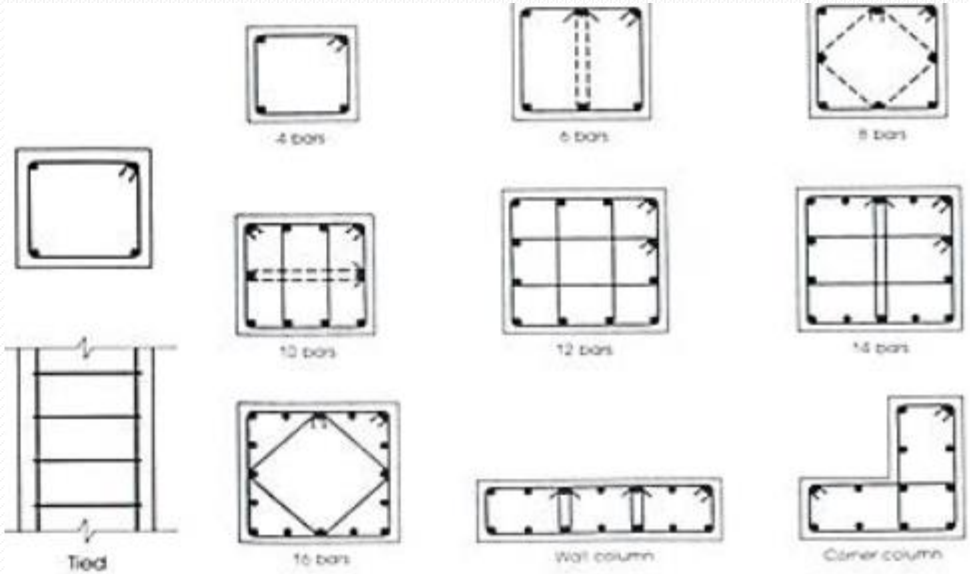
F. Classification on the basis of degree of slenderness; short column, slender column.

G. Classification on the basis of loading: axially loaded column, columns under uni-axial moment and columns under biaxial moment

TIED/SPIRAL COLUMNS

A. **Tied Columns:** Columns where main (longitudinal) reinforcements are held in position by separate ties spaced at equal intervals along the length.

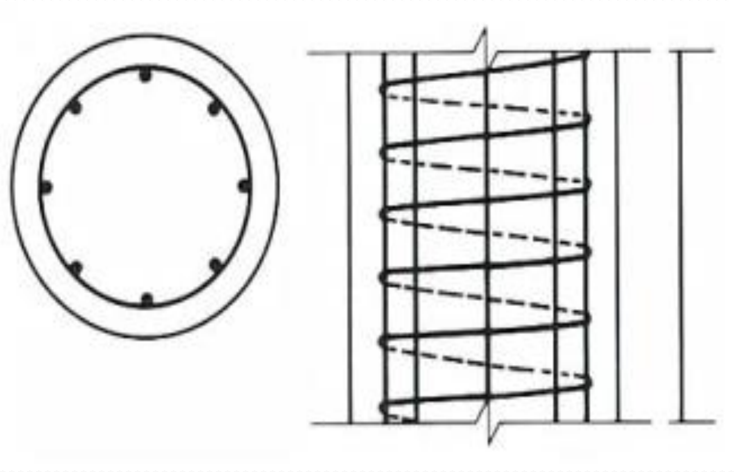
-And over 95% of all columns in buildings in non-seismic regions are tied columns.



Tied columns

Cont…

- b) **Spiral Columns:** Columns which are usually circular in cross section and longitudinal bars are wrapped by a closely spaced spiral.



Spiral columns

Behavior of Tied and Spiral columns

- The load deflection diagrams show the behavior of tied and spiral columns subjected to axial load.

- Because of the strength of spiral column enhanced by the tri axial stress resulting from the confinement of the core by the spiral reinforcement after spalling off concrete Ruther than buckling of reinforcement between ties, spiral columns are more ductile than tied columns.

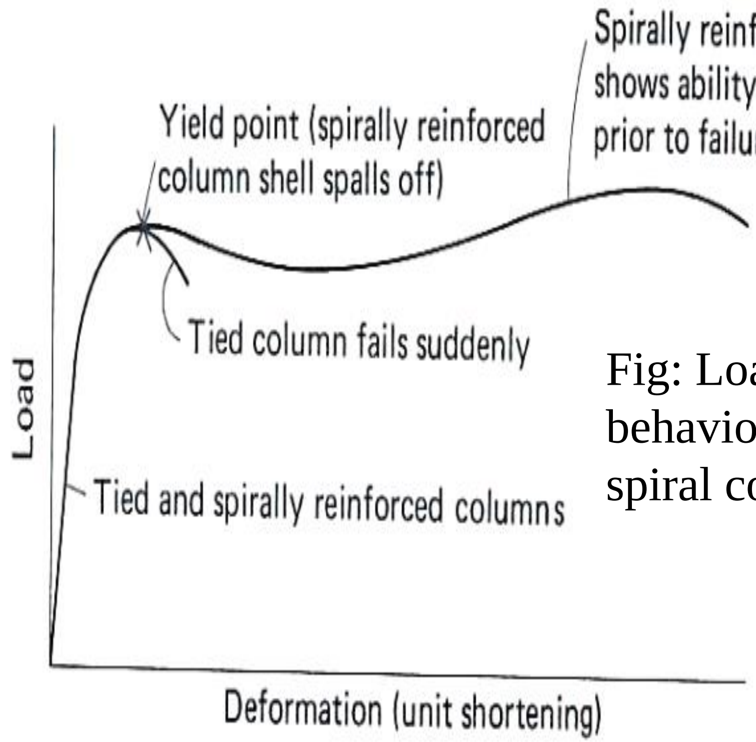


Fig: Load deflection behavior of tied and spiral columns

BRACED/UN-BRACED COLUMNS

- **a) Un-braced columns**

- Is one in which frames action is used to resist horizontal loads.

-The horizontal loads are transmitted to the foundations through bending action in the beams and columns which reduce their axial (vertical) load carrying capacity.

- Un-braced structures are generally quit flexible and allow horizontal displacement. When this displacement is sufficiently large to influence significantly the column moments, the structure is termed a sway frame.

Cont….

- **b) Braced columns:**

-Is braced against lateral loads using substantial bracing members such as shear walls, elevators, stairwell shafts, diagonal bracings or a combination of these.

-A column with in such a non-sway structure is considered to be braced and the second order moment on such column, P-Δ, is negligible.

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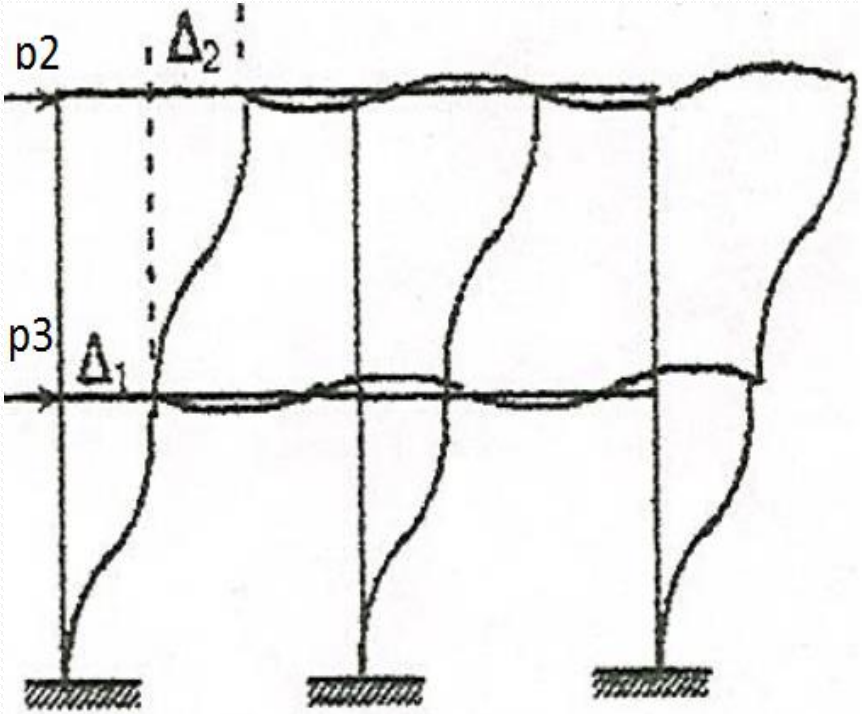
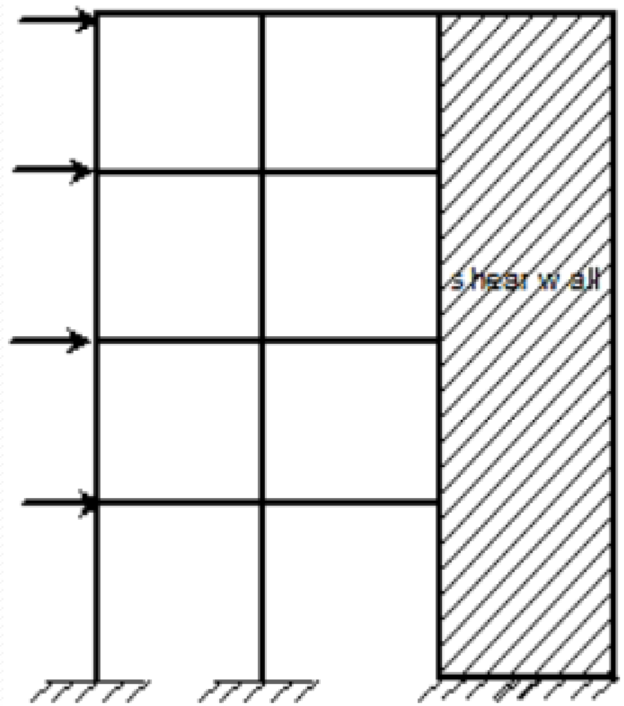


Sway Frame/ Un-braced columns



Non-sway Frame / Braced columns

SHORT/SLENDER COLUMNS

a) Short column

Has low slenderness ratio and their strengths are governed by the strength of the materials and the geometry of the cross section.

b) Slender columns

Has high slenderness ratio and their strength may be significantly reduced by lateral deflection.

Has additional moment due to deflection.

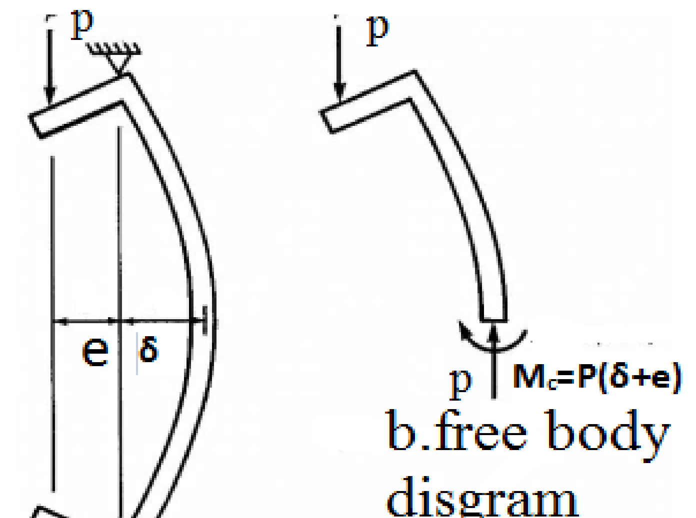
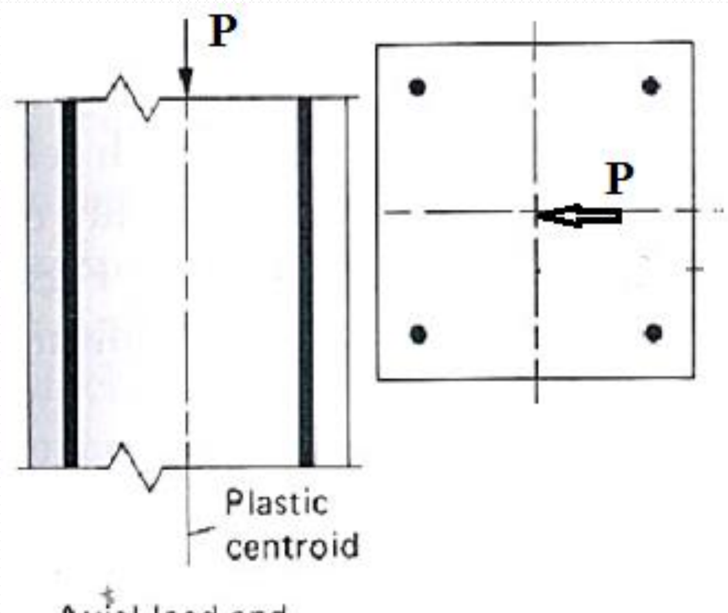


Fig:forces in slender column

CLASSIFICATION OF COLUMNS ON THE BASIS OF LOADING

1. **Axialy loaded columns**

- subjected to axial or concentric load without moments.



$$P_{do} = f_{cd}\left(A_g - A_{st}\right) + A_{st} f_{yd}$$

Where

A_g is gross concrete area

A_{st} is total reinforcement area

P_{do}-Ultimate capacity

Cont…

- 2. **Column under uni-axial bending**

-eccentricity is in only one direction so that moment is only about one axis.

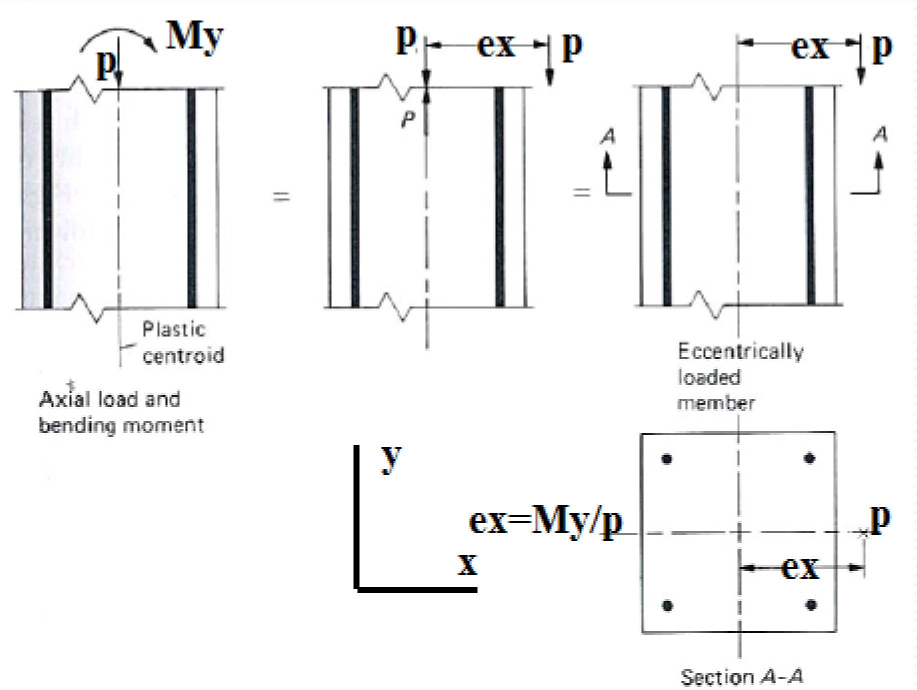


Fig:uni-axial bending

Cont…

- **Column under bi-axial bending**

-eccentricity is in both directions so that moment is about both axes.

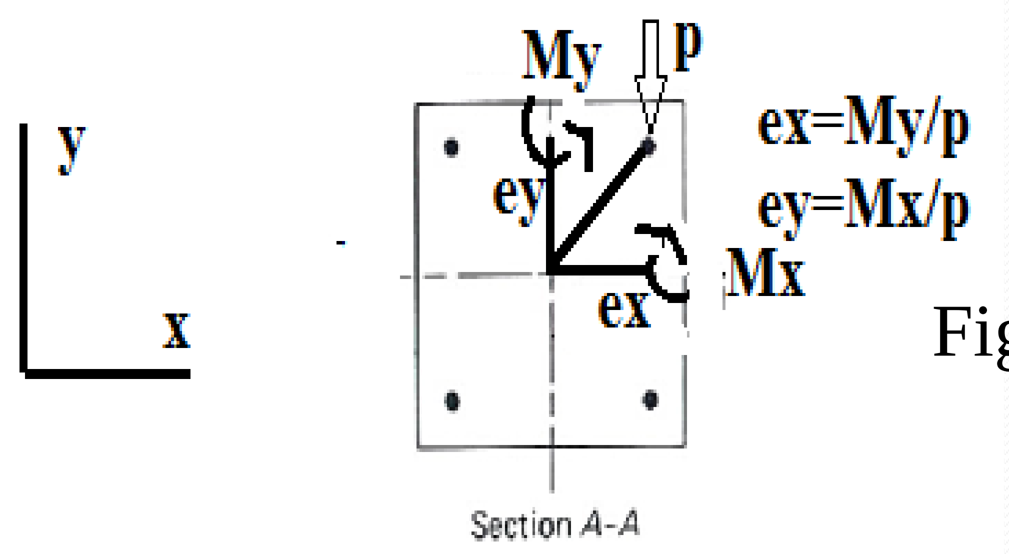


Fig:bi-axial bending

Interaction diagram

Let us consider idealized homogenous and elastic column with a compressive strength, fcu, equal to its tensile strength, ftu

$$f_{cu} = \frac{P}{A} + \frac{My}{I}$$

Dividing both sides by f_{cu}

$$1 = \frac{P}{f_{cu}A} + \frac{My}{f_{cu}I}$$

when M = 0, and is $P_{max} = f_{cu}A$.

when P=0 and is $M_{max} = f_{cu}I/y$

Substituting P_{max} and M_{max} gives

$$1 = \frac{P}{P_{max}} + \frac{M}{M_{max}}$$

Where

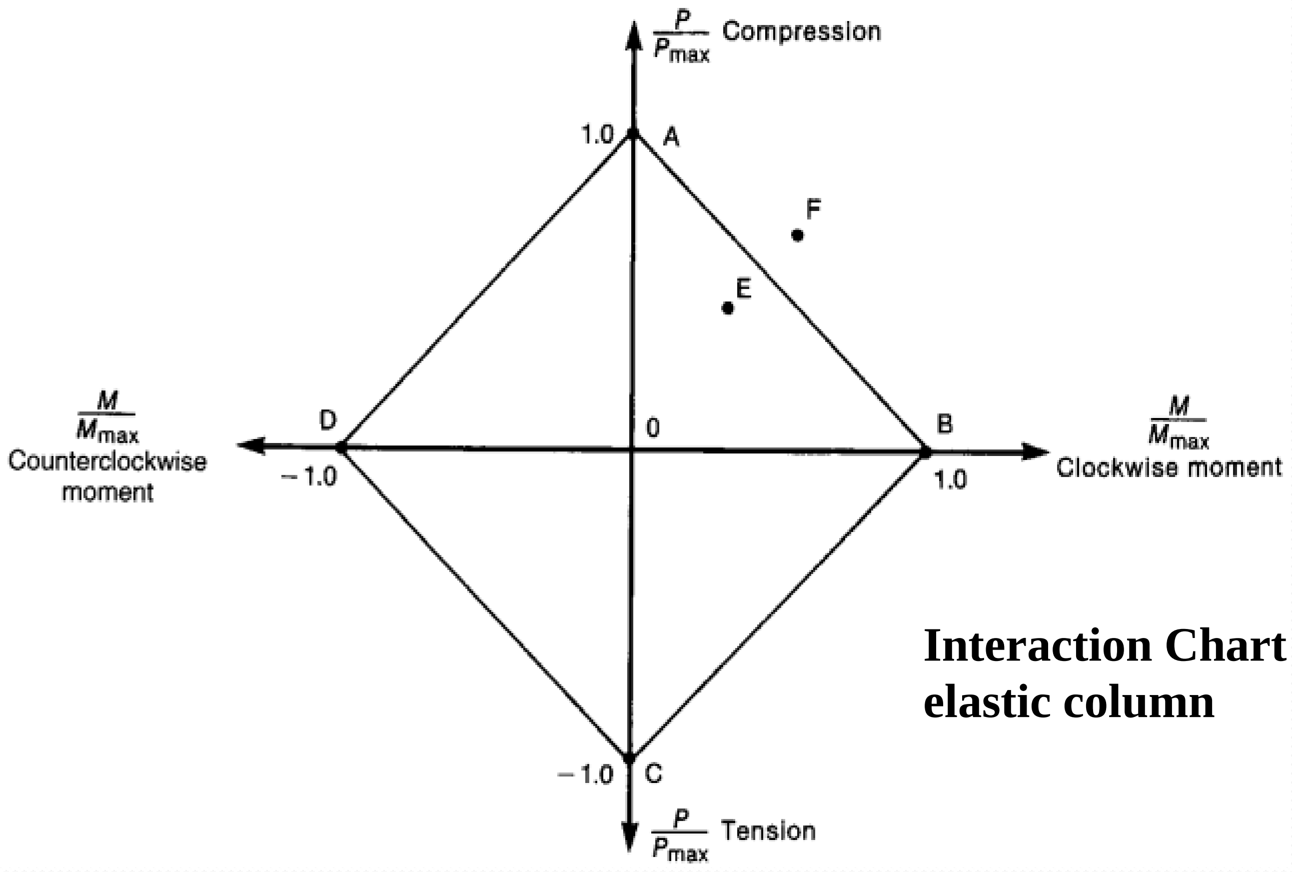
A, I -area and moment of inertia of the section

y -distance from the centroidal axis to the most highly compressed surface

P -Axial load, positive in compression

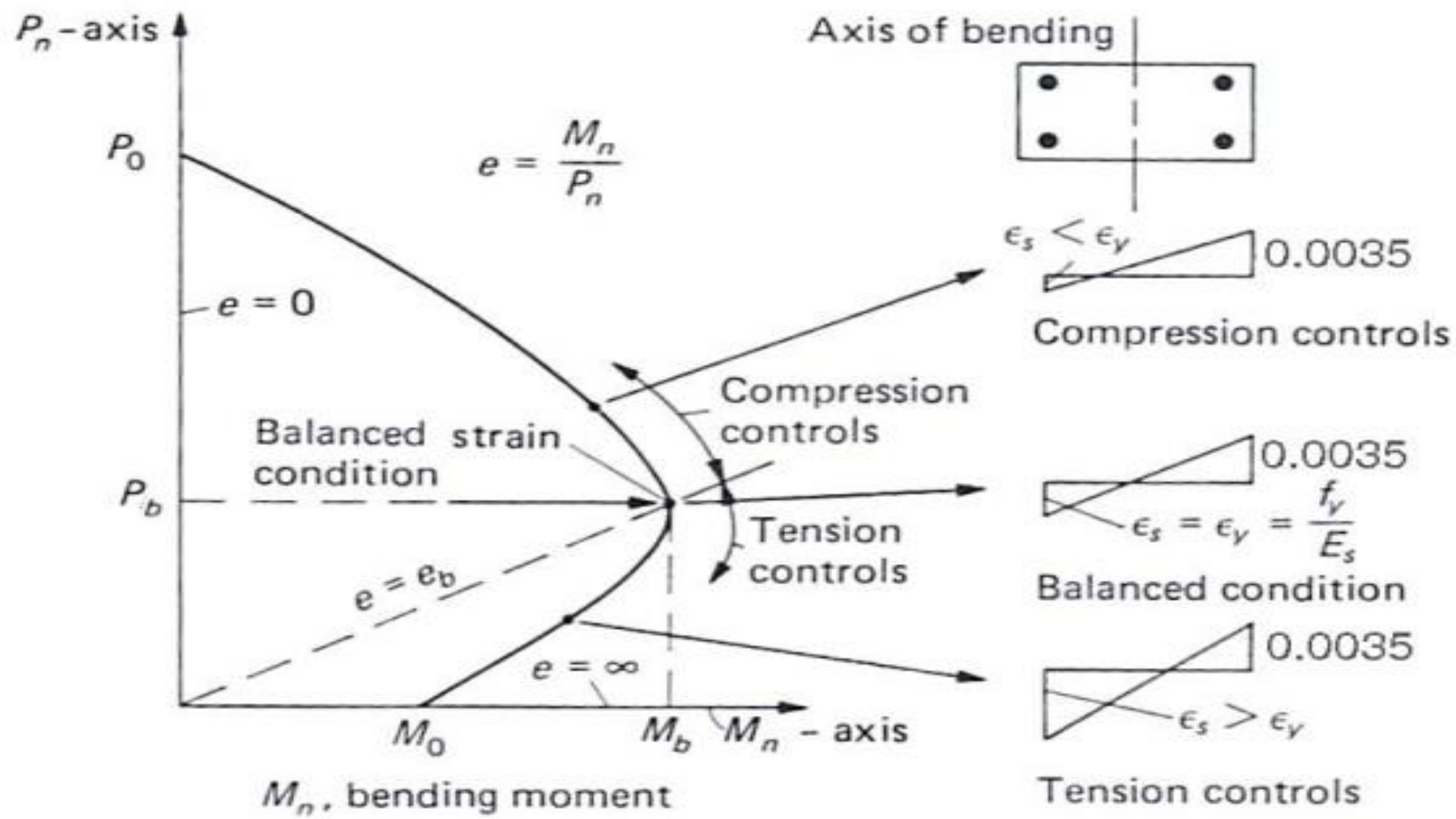
M- Moment, positive.

Cont…



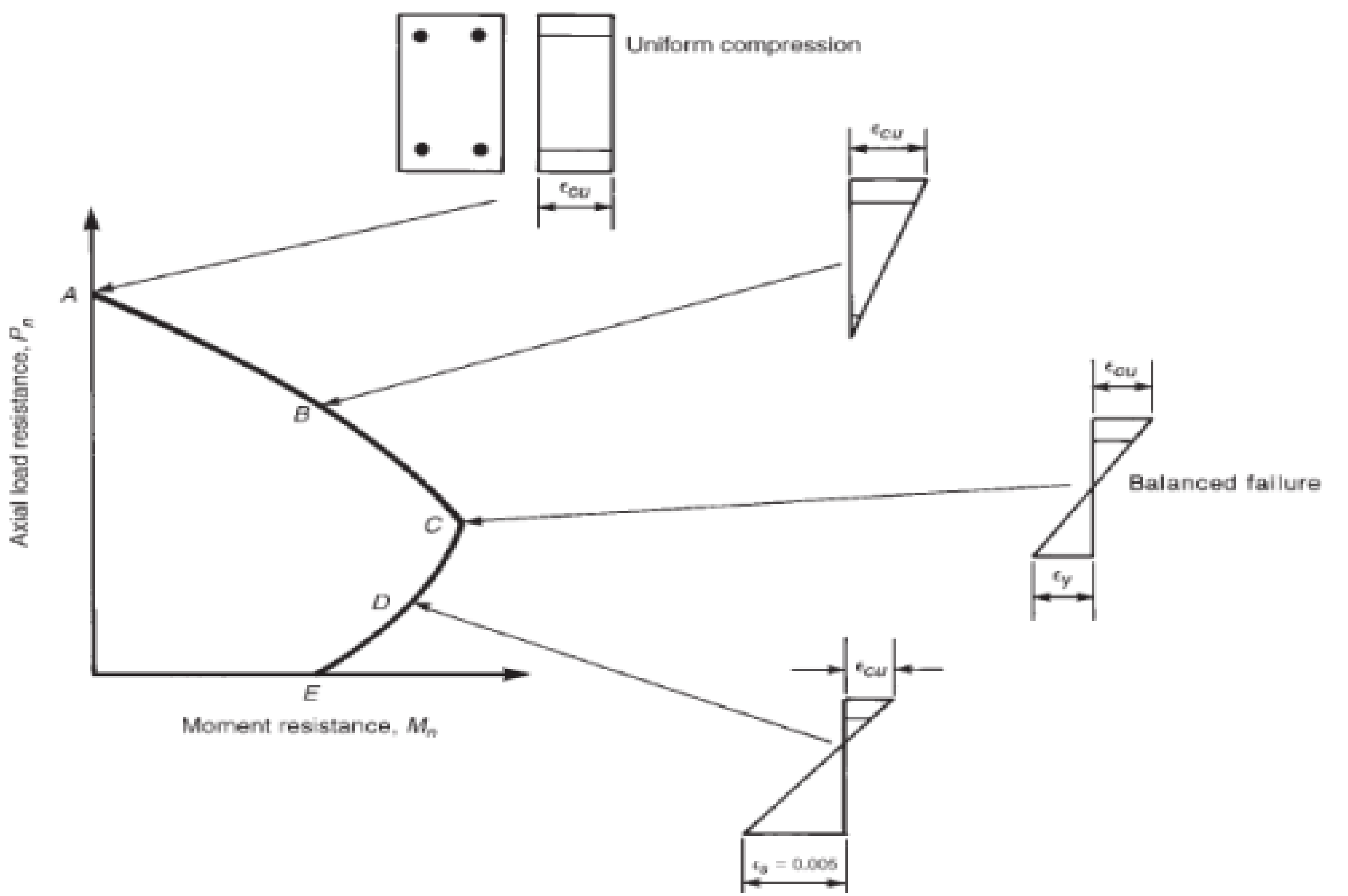
Interaction Chart for an elastic column

INTERACTION DIAGRAMS FOR REINFORCED CONCRETE COLUMNS



Interaction diagram for column in combined bending and axial load

Cont…



Strain distribution corresponding to points on the interaction diagram

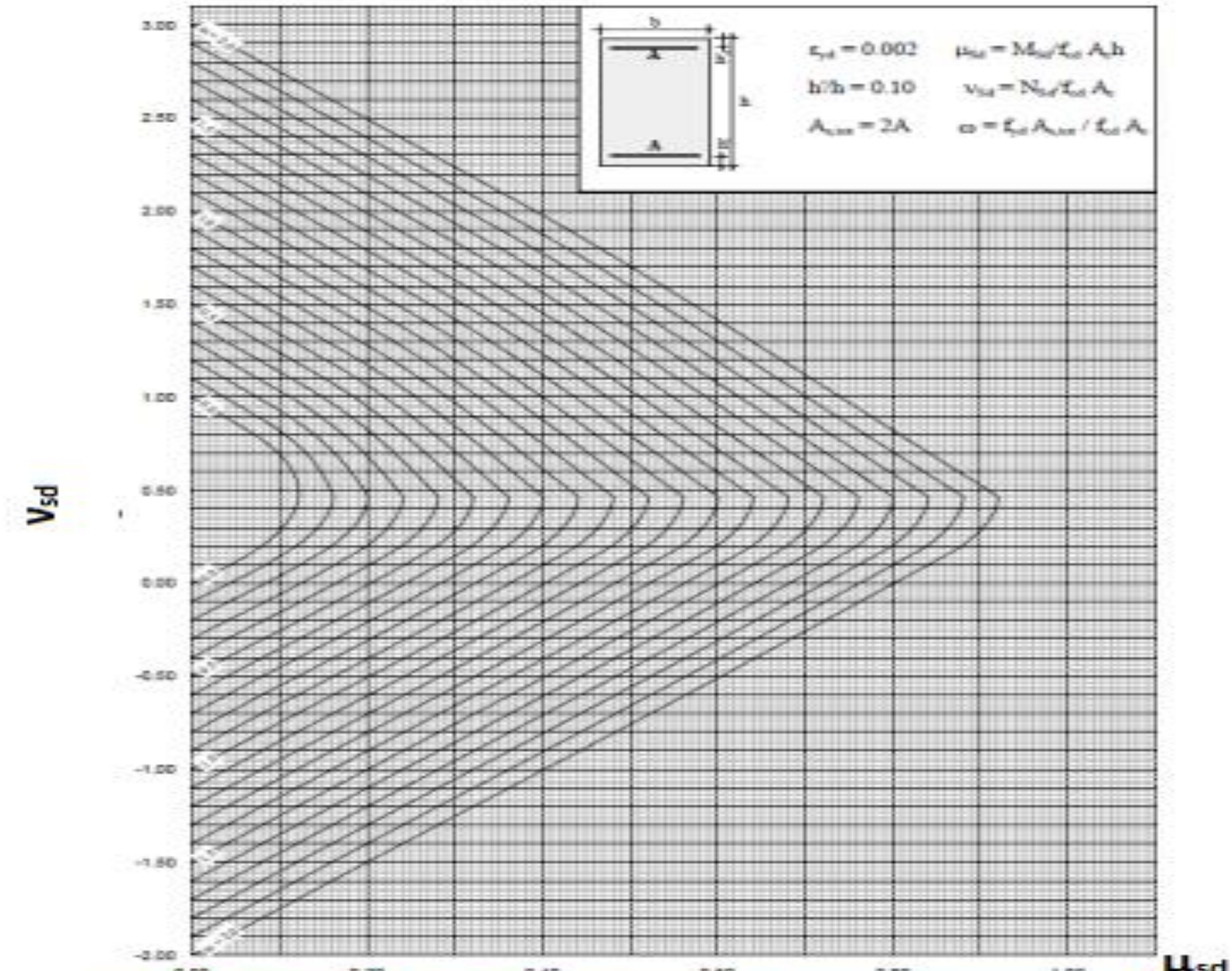
Using interaction chart

A. **uniaxial.**

- Assume a cross section, d' and evaluate d'/h to choose appropriate chart
  - Compute:
    - Normal force ratio: $\nu = N_u / f_{cd} bh$
    - Moment ratios: $\mu = M_u / f_{cd} bh^2$
  - Enter the chart and pick ω (the mechanical steel ratio), if the coordinate (ν, μ) lies within the families of curves. If the coordinate (ν, μ) lies outside the chart, the cross section is small and a new trail need to be made.
  - Compute
- $A_{s,tot} = \omega A_c f_{cd} / f_{yd}$
  - Check A_{tot} satisfies the maximum and minimum provisions
  - Determine the distribution of bars in accordance with the charts requirement .

Cont…

- uniaxial



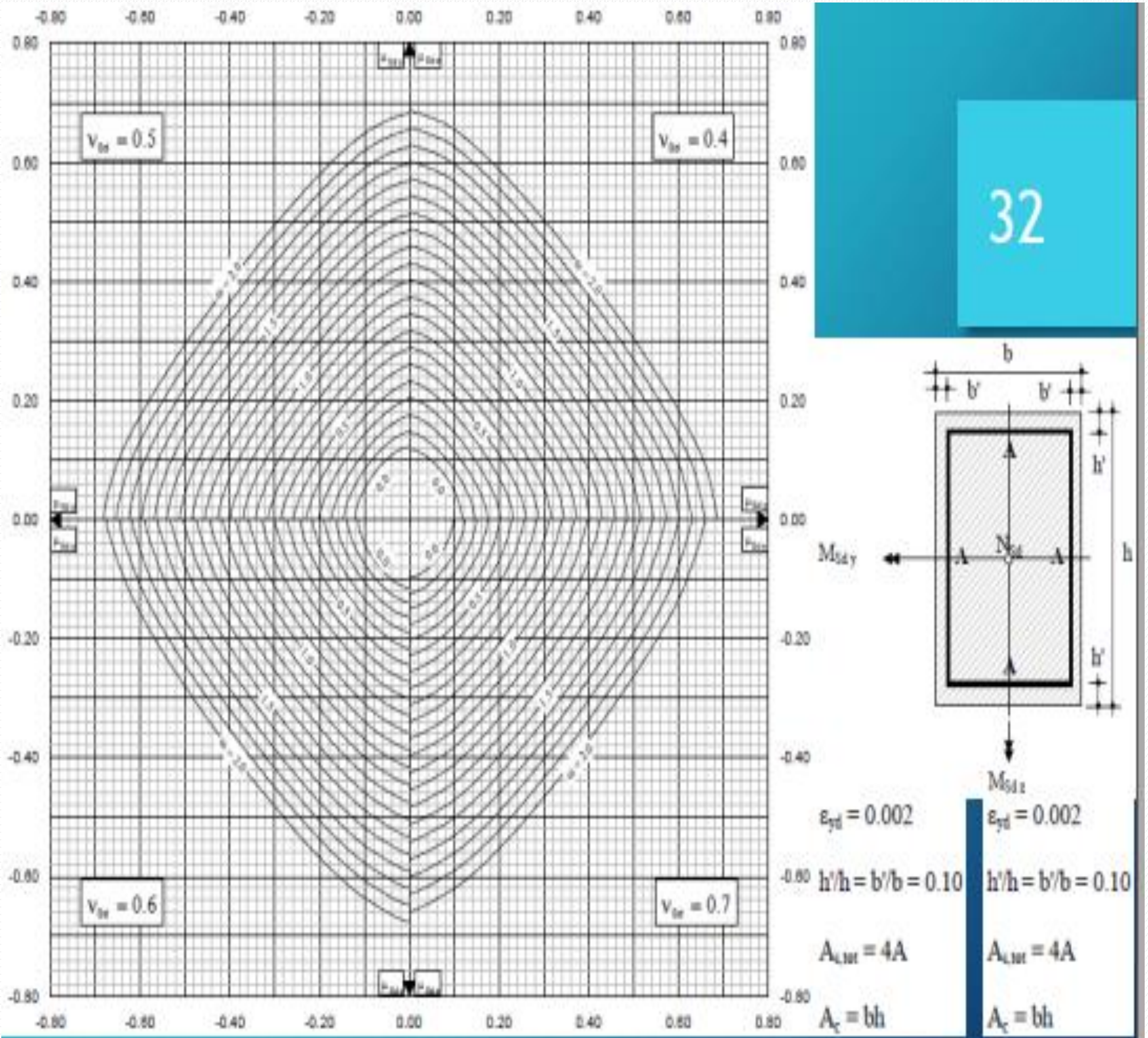
Cont…

B.biaxial.

- Select cross section dimensions h and b and also h’ and b’
  - Calculate h’/h and b’/b and select suitable chart
  - Compute: $\nu = N_u / f_{cd} bh$
    - Normal force ratio: $\mu_h = M_h / f_{cd} A_c h$ and $\mu_b = M_b / f_{cd} A_c b$
    - Moment ratios:
- Select suitable chart which satisfy and ratio:
  - Enter the chart to obtain ω
  - Compute $A_{s,tot} = \omega A_c f_{cd} / f_{yd}$
-
  - Check A_{tot} satisfies the maximum and minimum provisions
  - Determine the distribution of bars in accordance with the charts requirement .

Cont…

- Biaxial



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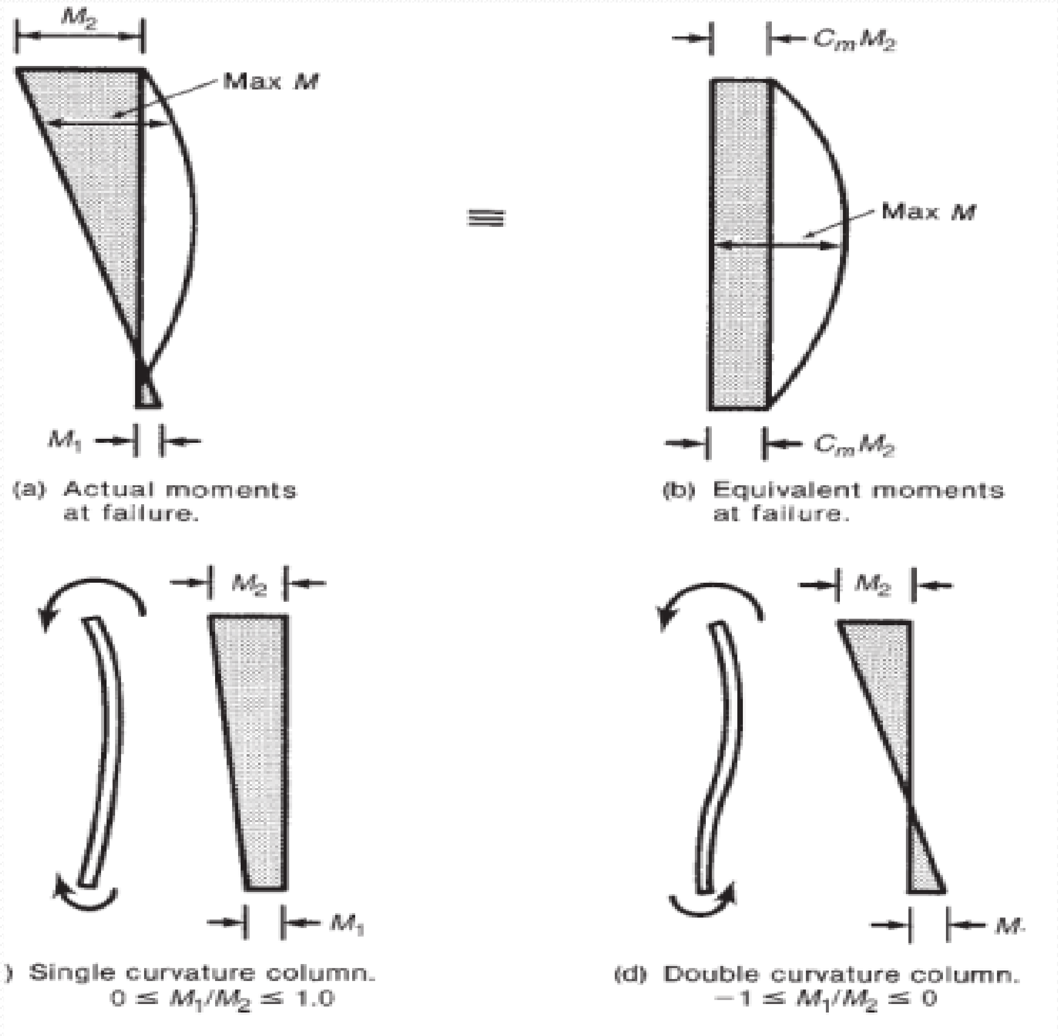


Equivalent moment factor

DESIGN OF COLUMNS ACCORDING TO ES EN 1992:2015

A. Geometric imperfections

Imperfections may be represented by an inclination, θi, given by:

$$\theta_i = \theta_0 \cdot \alpha_h \cdot \alpha_m$$

where

θ_0 is the basic value:

α_h is the reduction factor for length or height: $\alpha_h = 2/\sqrt{l};\ 2/3 \leq \alpha_h \leq 1$

α_m is the reduction factor for number of members: $\alpha_m = \sqrt{0.5(1+1/m)}$

l is the length or height [m],

m is the number of vertical members contributing to the total effect

Note: For use of θ_0 refer the National Annex. The recommended value for θ_0 is 1/200.

-Effect on isolated member: l = actual length of member, m =1.

- Effect on bracing system: l = height of building, m = number of vertical members contributing to the horizontal force on the bracing system.

- Effect on floor or roof diaphragms distributing the horizontal loads: l = story height, m =number of vertical elements in the story(s) contributing to the total horizontal force on the floor

cont…

- For isolated members , the effect of imperfections may be taken into account in two alternative ways:
- a) as an eccentricity, ei, given by: ei $=\theta i lo$ /2

Where *lo* is the effective length.

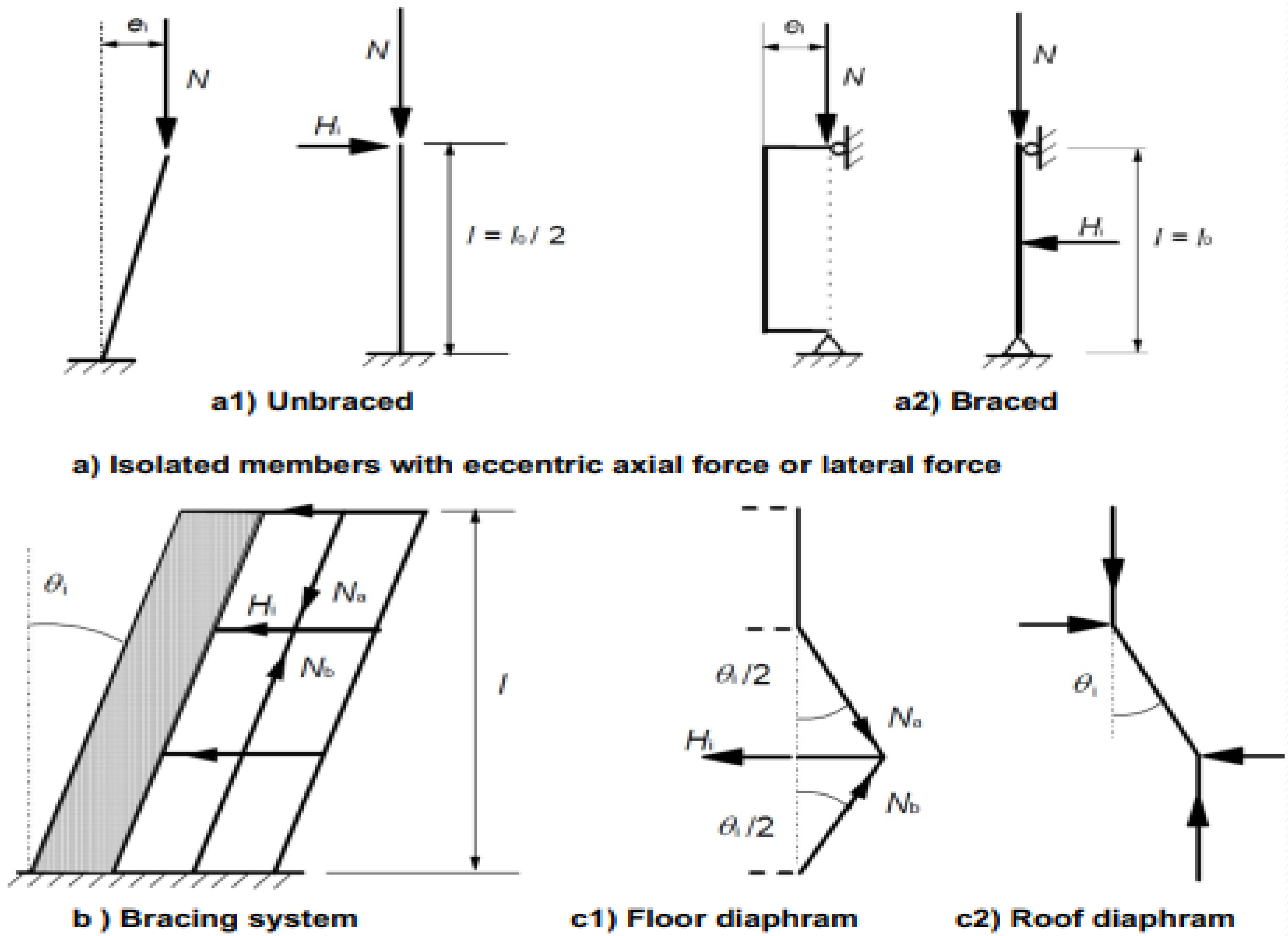
- For walls and isolated columns in braced systems, ei = *lo*/400 may always be used as a simplification, corresponding to αh = 1.
- b) as a transverse force, *Hi*, in the position that gives maximum moment:

For unbraced members, $Hi=\theta iN$

For braced members ,$Hi= 2\theta i\ N$

where N is the axial load. See next slide

Cont…



e_i
N
H_i
$l = l_0 / 2$
a1) Unbraced
e_i
N
H_i
$l = l_0$
a2) Braced
a) Isolated members with eccentric axial force or lateral force
θ_i
H_i
N_a
N_b
l
b) Bracing system
$\theta_i/2$
N_a
H_i
N_b
$\theta_i/2$
c1) Floor diaphram
θ_i
c2) Roof diaphram

Cont…

- For structures, the effect of the inclination �i may be represented by transverse forces, to be included in the analysis together with other actions.

Effect on bracing system, $H_i = \theta_i(N_b - N_a)$

Effect on floor diaphragm, $H_i = \theta_i(N_b + N_a)/2$

Effect on roof diaphragm, $H_i = \theta_i \times N_a$

where N_a and N_b are longitudinal forces contributing to H_i.

As a simplified alternative for walls and isolated columns in braced systems, an eccentricity $e_i = l_o/400$ may be used to cover imperfections related to normal execution deviations

Simplified criteria for second order effects

1.Slenderness criterion for isolated members:

second order effects may be ignored if the slenderness λ

- is below a certain value λ_{lim} , $\lambda_{lim} = \frac{20ABC}{\sqrt{n}}$

A = 1 / (1+0.2 φef)	(if φef is not known, A = 0.7 may be used)
B = 1+ 2ω	(if ω is not known, B = 1.1 may be used)
C =1.7 - rm	(if rm is not known C = 0.7 may be used)
φef -effective creep ratio	ω = $As$$f$yd/ ($Ac$$f$cd); mechanical reinforcement ratio; As is the total area of longitudinal reinforcement
n = NEd / ($Ac$$f$cd);	relative normal force
rm=M01/M02; moment ratio	; moment ratio

Cont…

- $M01$, $M02$ are the first order end moments, $|M02| \geq |M01|$
  In the following cases, r_m should be taken as 1.0 (i.e. C = 0.7):
  - for braced members in which the first order moments arise only from or predominantly due to imperfections or transverse loading
  - for unbraced members in general
  In cases with biaxial bending, the slenderness criterion may be checked separately for
  each direction.

Slenderness and effective length of isolated members

- The slenderness ratio is defined as follows: $\lambda = l_o / i$

where: l_o is the effective length, and i is the radius of gyration of the uncracked concrete section.

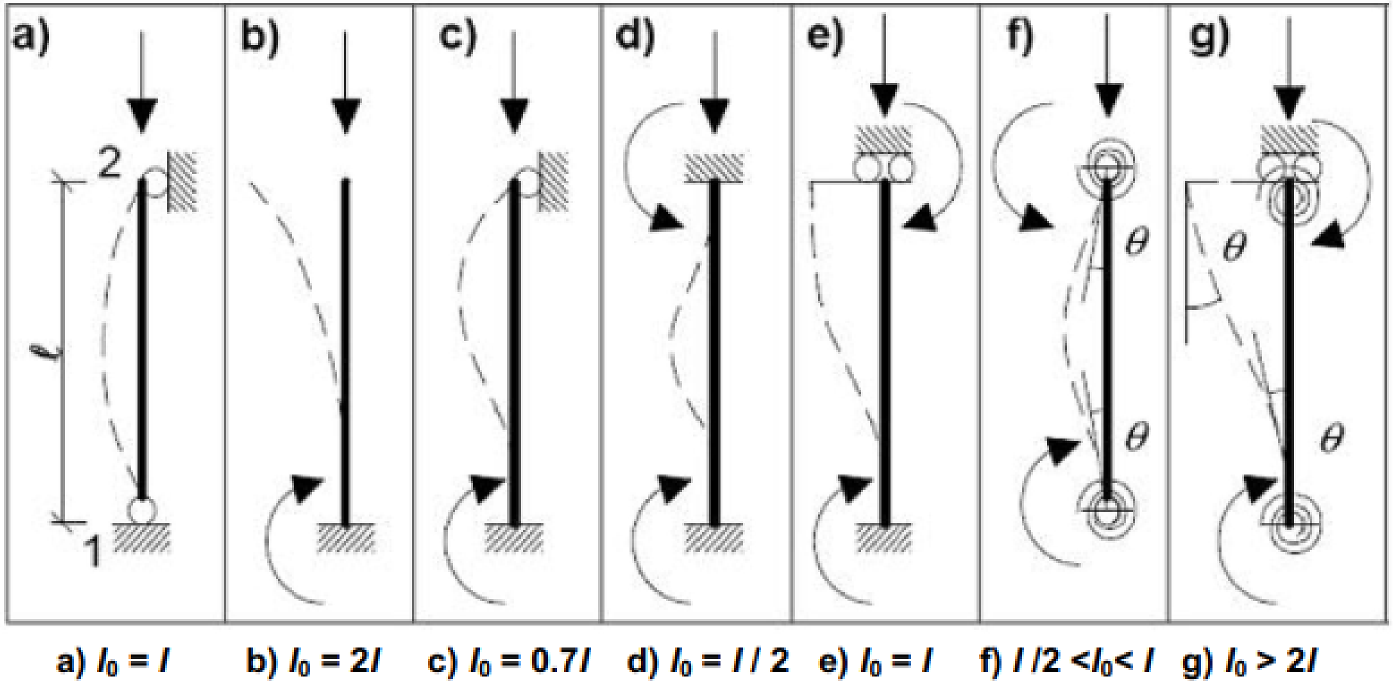


fig: Examples of different buckling modes and corresponding effective lengths for isolated members

However, for most real structures figures (f) and (g) only are applicable. They from real frame.

Cont...For detail read ES EN 1992-1-1_2015.section 5.8.3.2)

- For compression members in regular frames, the slenderness criterion should be checked with an effective length *lo* determined in the following way:

Braced members

$$l_o = 0.5l\sqrt{\left(1+\frac{k_1}{0.45+k_1}\right)\cdot\left(1+\frac{k_2}{0.45+k_2}\right)}$$

Unbraced members

$$l_o = l_{max}\left\{\sqrt{1+10\cdot\frac{k_1k_2}{k_1+k_2}}\ ;\left(1+\frac{k_1}{1+k_1}\right)\cdot\left(1+\frac{k_2}{1+k_2}\right)\right\}$$

where:

k1,k2 are the relative flexibilities of rotational restraints at ends 1 and 2 respectively. $k = (\theta/M)\cdot(EI/l)$

θ is the rotation of restraining members for bending moment *M* , *EI* is the bending stiffness of compression member.

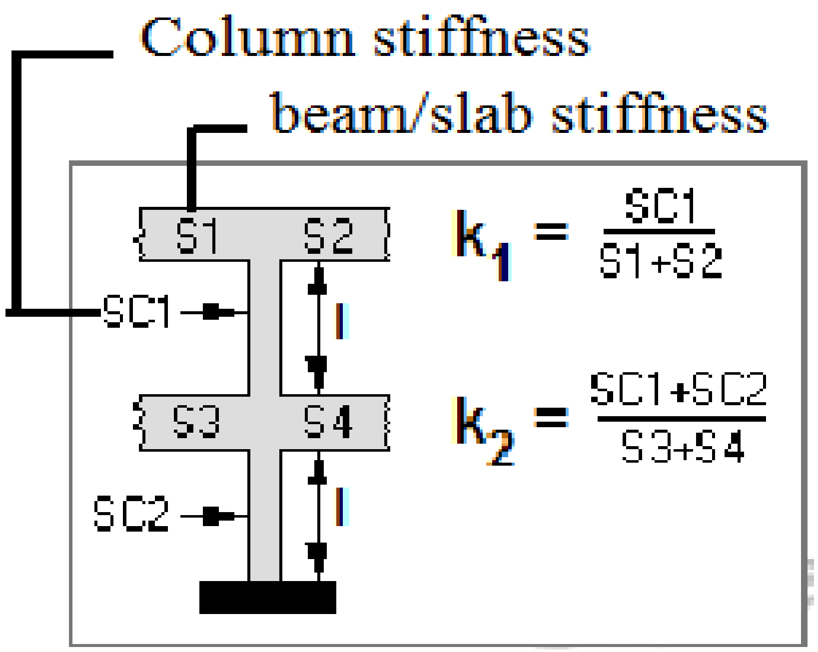
l is the clear height of compression member between end restraints.

Note: *k* = 0 is the theoretical limit for rigid rotational restraint, and *k* = ∞ represents the limit for no restraint at all. Since fully rigid restraint is rare in practice, a minimum value of 0.1 is recommended for k1 and *k*2

Cont…

Where θ is not known k1,k2 can be calculated from the ratio of the column bending stiffness to beam/slab bending stiffness, but taking only 50% of beam stiffness to allow for cracking .

Note: in this calculation of k1 and k2 only members properly framed into the end of the column in the appropriate plane of bending should be considered.



Creep

- The effect of creep shall be taken into account in second order analysis.
- The duration of loads may be taken into account in a simplified way by means of an effective creep ratio, φef

$\varphi_{ef} = \varphi(\infty,t_0) \cdot M_{0Eqp} / M_{0Ed}$

Where :$\varphi(\infty,t_0)$ is the final creep coefficient according to ES EN 1992-1-1_2015.section 3.1.4

M_{0Eqp} is the first order bending moment in quasi-permanent load combination (SLS)

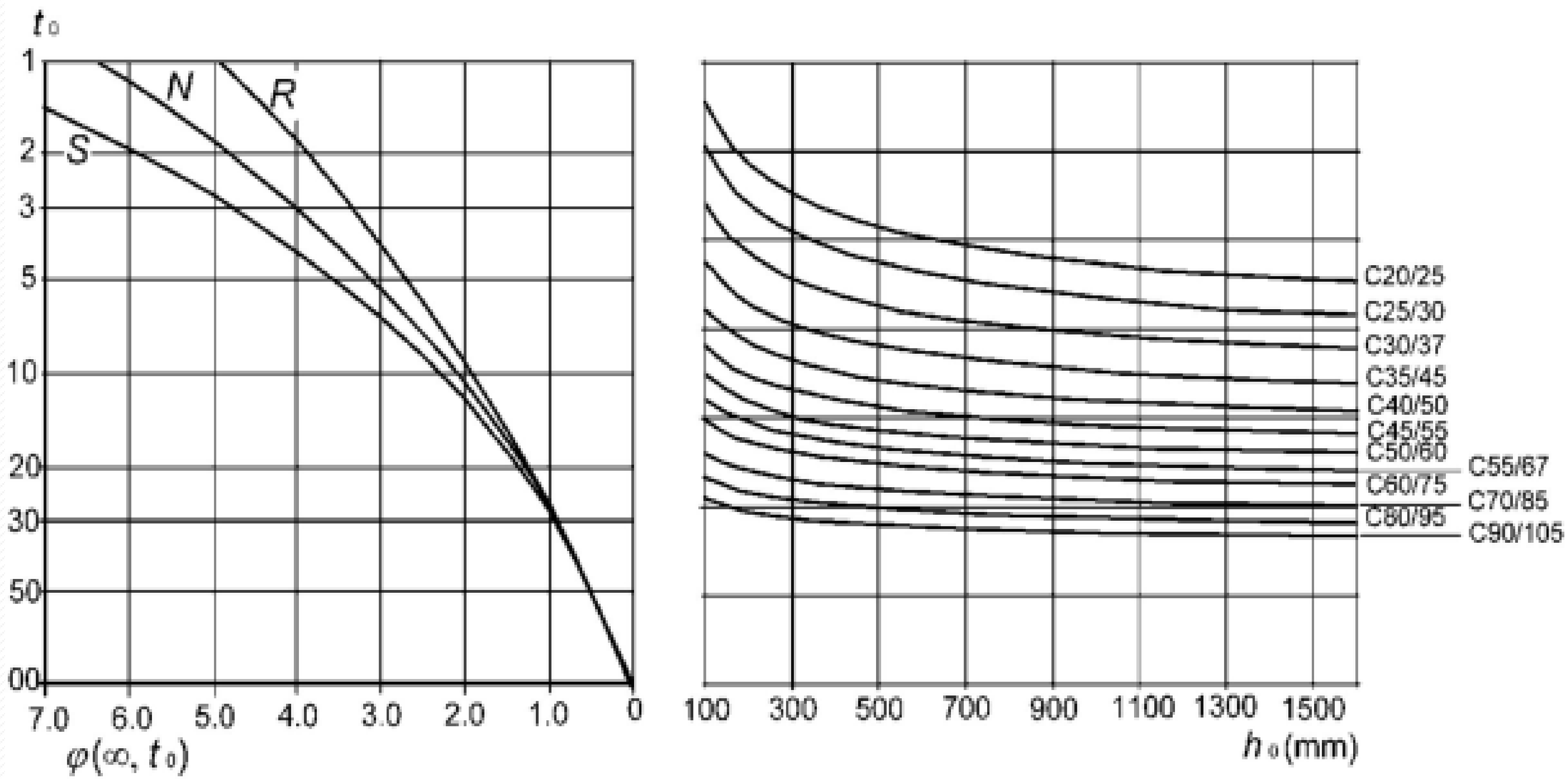
M_{0Ed} is the first order bending moment in design load combination (ULS)

Note: It is also possible to base φef on total bending moments M0Eqp and M0Ed, but this requires iteration
and a verification of stability under quasi-permanent load with $\varphi_{ef} = \varphi(\infty,t_0)$.

Cont…

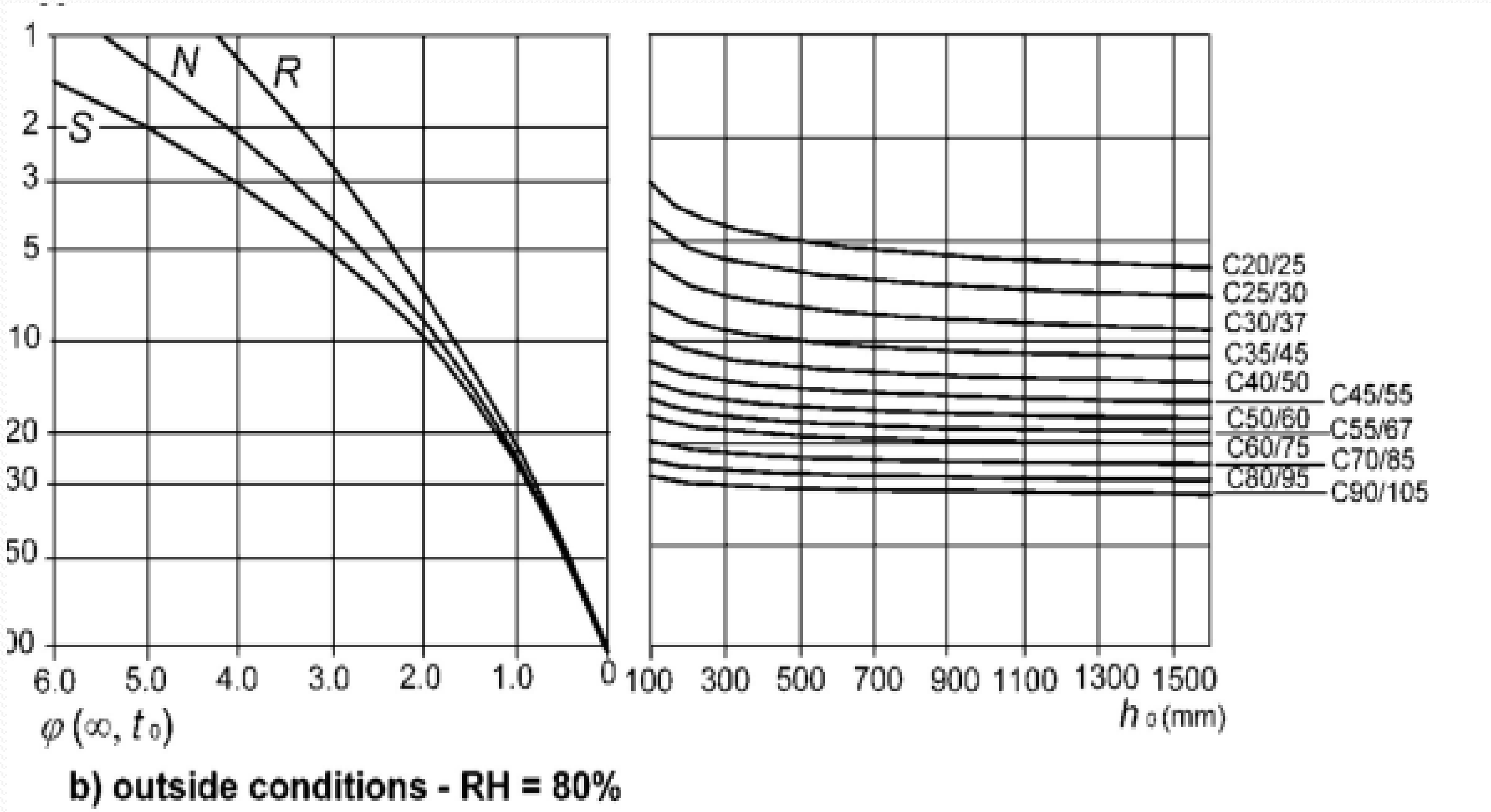
- If M_{0Eqp} / M_{0Ed} varies in a member or structure, the ratio may be calculated for the section with maximum moment, or a representative mean value may be used.
  -The effect of creep may be ignored, i.e. $\varphi_{ef} = 0$ may be assumed, if the following three conditions are met:
  - $\varphi(\infty,t_0) \leq 2$
  - $\lambda \leq 75$
  - $M_{0Ed}/N_{Ed} \geq h$
  -M_{0Ed} is the first order moment and h is the cross section depth in the corresponding direction

Determining creep coefficient(Refer ES EN 1992-1-1_2015.section 3.1.2 and 3.14)



a) **inside conditions - RH = 50%** RH-Relative humidity

Cont…

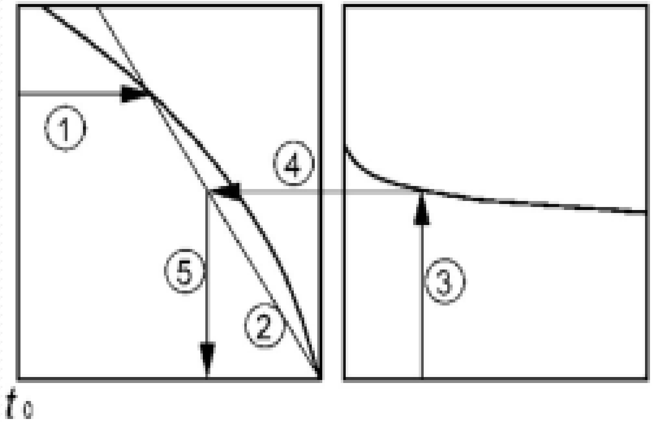


ıre 3.1: Method for determining the creep coefficient $\varphi(\infty, t_0)$ for concrete under

Cont…

cement of strength Classes CEM 42.5 R, CEM 52.5 N and CEM 52.5R Class R)
cement of strength Classes CEM 32.5 R, CEM 42.5 N (Class N)
cement of strength Classes CEM 32.5 N (Class S)

$\varphi(\infty, t_0)$ is the final creep coefficient
t_0 is the age of the concrete at time of loading in days
h_0 is the notional size = $2A_c/u$, where A_c is the concrete cross-sectional area and u is the perimeter of that part which is exposed to drying



Note:
- intersection point between lines 4 and 5 can also be above point 1
- for t_0 > 100 it is sufficiently accurate to assume t_0 = 100 (and use the tangent line)

Methods of analysis

(For detail read ES EN 1992-1-1_2015.section 5.8.5)

- The methods of analysis include a general method, based on non-linear second order analysis and the following two simplified methods:

(a) Method based on nominal stiffness

(b) Method based on nominal curvature

a).Method based on nominal stiffness

-In a second order analysis based on stiffness,nominal values of the flexural stiffness should be used, taking into account the effects of cracking, material non-linearity and creep on the overall behavior.

Nominal stiffness

- The following model may be used estimate the nominal stiffness of slender compression members with arbitrary cross section.
- $EI = K_c E_{cd} I_c + K_s E_s I_s$

where:
E_{cd} is the design value of the modulus of elasticity of concrete
I_c is the moment of inertia of concrete cross section
E_s is the design value of the modulus of elasticity of reinforcement
I_s is the second moment of area of reinforcement, about the centre of area of the concrete
K_c is a factor for effects of cracking, creep
K_s is a factor for contribution of reinforcement

Cont…

- *The following factors may be used in Expression EI calculation above, provided $p \geq 0.02$*

$K_s = 1$ $\quad\quad\quad$ $K_c = k_1 k_2 / (1+\varphi_{ef})$

Where:

ρ - is the geometric reinforcement ratio, A_s/A_c

A_s- is the total area of reinforcement $\quad$ A_c- is the area of concrete section

φ_{ef}- is the effective creep ratio

k_1- is a factor which depends on concrete strength class

k_2- is a factor which depends on axial force and slenderness

$k_1 = \sqrt{f_{ck}/20}$ (MPa) $\quad\quad$ $k_2 = n\frac{\lambda}{170} \leq 0.2$

where λ - is the slenderness ratio

n is the relative axial force, $N_{Ed} / (A_c f_{cd})$

if the slenderness ratio λ is not defined k2 may be taken as

$k_2 = n x 0.3 \leq 0.2$

Cont…

As a simplified alternative, provided $p \geq 0.01$, the following factors may be used in Expression of EI calculation

$K_s = 0$

$K_c = 0.3\ (1 + 0.5\varphi_{ef}))$.

In statically indeterminate structures, unfavourable effects of cracking in adjacent members should be taken into account

The stiffness should be based on an effective concrete modulus:

$E_{cd,eff} = E_{cd}\ (1+\varphi_{ef}\ )$

E_{cd} is the design value of the modulus of elasticity according to 5.8.6 (3)

φ_{ef} is the effective creep ratio; same value as for columns may be used

Moment magnification factor

- First order Moment M_{0Ed} is magni $M_{Ed} = M_{0Ed}\left[1 + \frac{\beta}{(N_B/N_{Ed}) - 1}\right]$

M_{0Ed} is the first order moment

β is a factor which depends on distribution of 1st and 2nd order moments,

N_{Ed} is the design value of axial load

N_B is the buckling load based on nominal stiffness

For isolated members with constant cross section and axial load, the second order moment may normally be assumed to have a sine-shaped distribution. Then

$\beta = \frac{\pi^2}{c0}$ where:

$c0$ is a coefficient which depends on the distribution of first order moment (for instance, $c0 = 8$ for a constant first order moment, $c0 = 9.6$ for a parabolic and 12 for a symmetric triangular distribution etc.)

Cont…

- For members without transverse load, differing first order end moments M01 and M02 may be replaced by an equivalent constant first order moment M0e according to ES EN 1992-1-1_2015.section 5.8.8.2 (2).
- Consistent with the assumption of a constant first order moment, c0= 8 should be used.
- The value of c0 = 8 also applies to members bent in double curvature.

If the above conditions are not applicable β=1 is used and then

$$M_{Ed} = \frac{M_{0Ed}}{1-(N_{Ed}/N_B)}$$

Cont…

b)Method based on nominal curvature

- The design moment is:

$M_{Ed} = M_{0Ed} + M_2$

Where:

M_{0Ed} is the 1st order moment, including the effect of imperfections.

M_2 is the nominal 2nd order moment.

The maximum value of M_{Ed} is given by the distributions of M_{0Ed} and M_2; the latter may be taken as parabolic or sinusoidal over the effective length.

- For statically indeterminate members, M_{0Ed} is determined for the actual boundary conditions, where as M_2 will depend on boundary conditions via the effective length. For members without loads applied between their ends, differing first order end moments M_{01} and M_{02} may be replaced by an equivalent first order end moment M_{0e}:

$$\mathbf{M_{0e} = 0.6M_{02} + 0.4M_{01} \geq 0.4M_{02}}$$

M_{01} and M_{02} should have the same sign if they give tension on the same side, otherwise opposite signs. Furthermore, $|M_{02}| \geq |M_{01}|$.

Cont…

- The nominal second order moment $M2$ in Expression.

$M2 = N\mathrm{Ed}e2$

where:

Ned is is the design value of axial force

$e2$ is the deflection = $(1/r)l_0^2)/c$

$1/r$ is the curvature, lo is the effective length,

c is a factor depending on the curvature distribution

For constant cross section, c = 10 ($\approx\pi2$) is normally used. If the first order moment is constant, a lower value should be considered (8 is a lower limit, corresponding to constant total moment).

Curvature

- For members with constant symmetrical cross sections (incl. reinforcement), the following may be used:

$1/r = K_r \cdot K_\varphi \cdot 1/r_0$

Where:

K_r is a correction factor depending on axial load,

K_φ is a factor for taking account of creep

$1/r_0 = \varepsilon_{yd}/(0.45\, d)$

$\varepsilon_{yd} = f_{yd}/E_s$

d is the effective depth

Cont…

- If all reinforcement is not concentrated on opposite sides, but part of it is distributed parallel to the plane of bending, d is defined as:

$d = (h/2) + i_s$

where i_s is the radius of gyration of the total reinforcement area

kr should be taken as:

$K_r = (n_u - n)\ (n_u - n_{bal}) \leq 1$

$n = N_{Ed} / (A_c f_{cd})$, relative axial force

N_{Ed} is the design value of axial force

$n_u = 1 + \omega$

n_{bal} is the value of n at maximum moment resistance; the value 0.4 may be used

$\omega = A_s f_{yd} / (A_c f_{cd})$

A_s is the total area of reinforcement

A_c is the area of concrete cross section

Cont…

- The effect of creep should be taken into account by the following factor:

$K\varphi = 1 + \beta\varphi ef \geq 1$

where:

φef is the effective creep ratio,

$\beta = 0.35 + f ck/200 - \lambda/150$

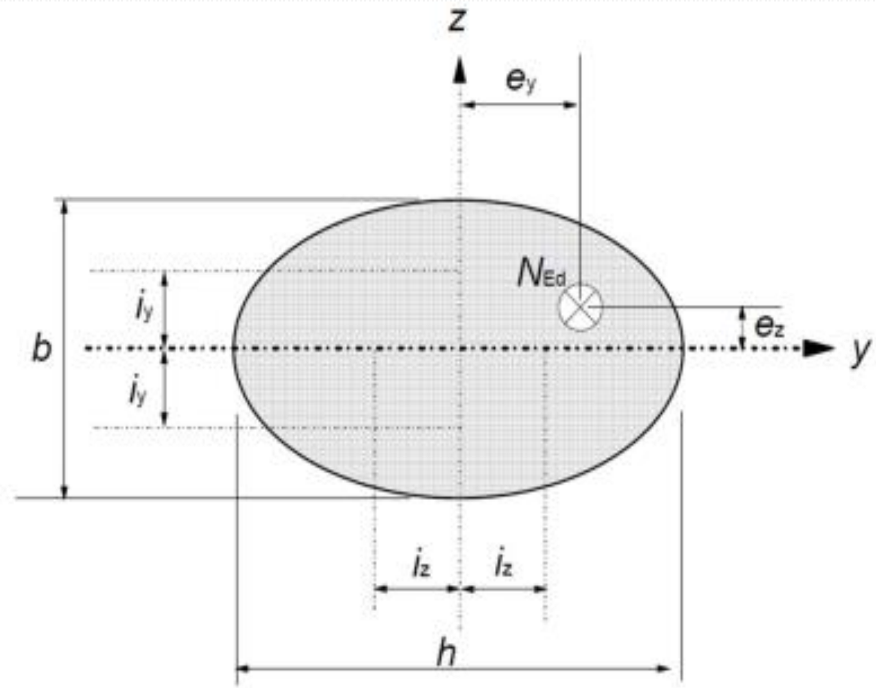
λ is the slenderness ratio

Biaxial bending

- No further check is necessary if the slenderness ratios satisfy the following two conditions:
  $\lambda y / \lambda z \leq 2$ and $\lambda z / \lambda y \leq 2$

and if the relative eccentricities ey/heq and ez/beq (in the Figure shown) satisfy one the following conditions:

- $\frac{(\frac{ey}{heq})}{(\frac{ez}{beq})} \leq 0.2 \; or \; \frac{\left(\frac{ez}{beq}\right)}{\left(\frac{ey}{heq}\right)} \leq 0.2$



Definition of eccentricities e_y and e_z

Cont…

Where:

b, h	are the width and depth of the section
b_{eq}	$= i_y \cdot \sqrt{12}$ and $h_{eq} = i_z \cdot \sqrt{12}$ for an equivalent rectangular section
λ_y, λ_z	are the slenderness ratios l_0/i with respect to y- and z- axis respectively
i_y, i_z	are the radii of gyration with respect to y –and z –axis respectively
e_z	$= M_{Edy} / N_{Ed}$; eccentricity along z-axis
e_y	$= M_{Edz} / N_{Ed}$; eccentricity along y-axis
M_{Edy}	is the design moment about y-axis, including second order moment
M_{Edz}	is the design moment about z-axis, including second order moment
N_{Ed}	is the design value of axial load in the respective load combination

Cont…

- In the absence of an accurate cross section design for biaxial bending, the following simplified criterion may be used.

$$\left(\frac{M_{Edz}}{M_{Rdz}}\right)^{a} + \left(\frac{M_{Edy}}{M_{Rdy}}\right)^{a} \leq 1.0$$

Where:

$M_{Edz/y}$ is the design moment around the respective axis, including a 2nd order moment.

$M_{Rdz/y}$ is the moment resistance in the respective direction

a is the exponent;
for circular and elliptical cross sections: $a = 2$
for rectangular cross section

N_{Ed}/N_{Rd}	0.1	0.7	1.0
a =	1.0	1.5	2.0

with linear interpolation for intermediate values

N_{Ed} is the design value of axial force

N_{Rd} $= A_c f_{cd} + A_s f_{yd}$, design axial resistance of section.
where:
A_c is the gross area of the concrete section
A_s is the area of longitudinal reinforcement

Reinforcement for column

- **Longitudinal reinforcement**

-Longitudinal bars should have a diameter of not less than ϕ_{min}. The recommended value is **8 mm.**

-The total amount of longitudinal reinforcement should not be less than *A*s,min . The recommended value is given by

As,min=0.1NEd/fyd or 0.002Ac

fyd is the design yield strength of the reinforcement

NEd is is the design axial compression force

-The area of longitudinal reinforcement should not exceed *As,max*

The recommended value is **0.04 Ac** outside lap locations unless it can be shown that the integrity of concrete is not affected, and that the full strength is achieved at ULS. This limit should be increased to 0.08 Ac at laps.

Cont…

- **Transverse reinforcement**
  The diameter of the transverse reinforcement (links, loops or helical spiral reinforcement) should not be less than **6 mm or one quarter of the maximum diameter of the longitudinal bars**, whichever is the greater.
- The diameter of the wires of welded mesh fabric for transverse reinforcement should **not be less than 5 mm.**
  The transverse reinforcement should be anchored adequately.
  The spacing of the transverse reinforcement along the column should not exceed $s_{cl,tmax}$
- The recommended value $s_{cl,tmax}$ of is the least of the following three distances:
  - **20 times the minimum diameter of the longitudinal bars**
  - **the lesser dimension of the column**
  - **400 mm**